WHY WE MULTIPLY BETTING ODDS - AN ECONOMETRIC AND PROBABILISTIC APPROACH

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Abstract: This paper provides one interpretation for a fact that betting odds are multiplied in a case when a bettor accumulates bets on more than one game. There are many reasons and explanations in literature why this happens and why this is, from a probability theory point of view, a fair thing to do. We will interpret it by making a fair assumption that this should be a zero sum game, i.e. long term profit for both sides, the bettor and the bookmaker equals to a zero. We will also remind and show that a zero sum game assumption is equivalent to a bookmaker multiplication of betting odds when bettor makes a bet on multiple games. These concepts are known, but on the other hand our aim is to provide a simple overview of these concepts.

Additionally, we will offer an interpretation where, mathematically speaking, bettor and bookmaker are treated equally. We will investigate how a multiplication rule looks like from a perspective of a bookmaker when being considered as a bettor and vice versa. This will give us an insight for a slightly different multiplication rule.

Keywords: Betting Odds, Zero Sum Assumption, Expectations

1 Introduction

We analyze betting where one bettor is trying to guess an outcome of a sport game betting on just one game. Also, a case where bettor accumulates multiple games trying to guess outcome of more than one game will be mentioned. Rules for betting on sport events follow basic laws of probability. If a bettor bets with 10 EUR on an outcome of one game (win, loss or tie) with odds 1.35 then, in case that his guess was correct, the bettor will win $10 \times 1.35 = 13.5$ EUR.

If, for example, a bettor puts 10 EUR bet trying to guess outcomes of two games with odds 1.35 and 1.55, then ,in case his guess is correct, he is going to win $10 \times 1.35 \times 1.55 = 20.925$ EUR. But, in case that bettor miss at least one of those two matches, he will lose all 10 EUR. Main topic of this paper is to provide a simple interpretation of a 'multiplication rule' for odds when someone is betting on accumulated sport events.

At this point we need to emphasize that a calculation above doesn't include a cost for running the market. This cost is paid by the bettor to a bookmaker. Usually, it is 5 %. That is inbuilt profit for a bookmaker.

We firstly introduce a random variable that describes the profit that a player makes after placing a bet. Let us define D_P (P stands for a 'player' or bettor) as a random variable that describes the gain for a player. Let U be the initial investment made. Let p be the probability that a player will win. Clearly, estimation of p might be biased. It is known that estimation of probability p is a hard process. But, nevertheless p is correlated with odds that are, in general, estimation that a bookmaker makes based on previous games, statistics of the league or championship. Also, let us define a number k that represents the betting odd, where $k > 1$. If $D_P = -U$, then it means that a bettor didn't have a wining ticket. His prediction of an outcome of a game wasn't correct. Then the probability for that event is given by $P(D_P = -U) = 1 - p$. In the case that bettor made a correct prediction, a prize for that will be kU, so a clean profit is $kU - U$ (since a bettor placed a bet for amount U). Therefore, $P(D_P = kU - U) = p$. So, finally we put a random variable

D_P as:

$$D_P = \begin{pmatrix} -U & kU - U \\ 1-p & p \end{pmatrix} \tag{1}$$

We will also introduce a random variable D_B (where B stands for a bookmaker). It will also have two different values like D_P. More details will follow in the next section.

There is always the question of how to define a fair game. Taking everything into consideration, a fair game is usually defined as the one in which the expected newly created surplus is zero for both sides involved in the game, the bettor and the bookmaker. We call this a *zero sum assumption.* Formally speaking, we can put

$$E(D_P) = E(D_B) = 0 \tag{2}$$

where $E(D_P)$ and $E(D_B)$ are expectations of random variables D_P and D_B.

Also, we need to underline that taxes and other betting fees are not included in the calculations that will follow. We omit those for the sake of simplicity.

Estimation of odds shows to be a difficult problem. For example see Bird, R. McCrae, M. (1987). More on modeling of betting odds could be found in Dixon, M. J. Coles, S. G. (1997). and Figlewski, S. (1979). On the other hand, this paper deals with one type of interpretation of betting odds assuming that odds are direct consequence of probability of certain outcome of a game. Further interpretation on market efficiency and odds setting could be found in Forrest, D., Goddard, J. Simmons, R. (2005). and Pankoff, L. D. (1968).

2 Zero sum assumption and probabilities

It is important to underline once again that we don't have an intention to model probabilities of certain outcomes of sport events. Betting odds are subjective estimations made by experts that provide services for a bookmaker. Their estimations are biased and based upon statistics from previous events. But nevertheless it is a great challenge to estimate those accurately. Another motive that might guide bookmakers decision is how many bets are made on a certain event. That require additional hedging from a bookmaker side, so that risk of loosing money is minimized. But in ideal case, we would like to interpret connection between odds and probability based on an assumption of a fair game. Another point is, to what extent we may call it a fair game given the reality of hedging, biased estimation of probability etc.

The first result that we present deals with the case when bettor makes a bet on just one game with an odd k. A probability that a bettor will make profit is denoted by p. To be more specific, we have:

Theorem 2.1. *Let k be a betting odd and let p be a probability of a win after a bet. Then, a zero sum assumption is equivalent to $k = 1/p$.*

Proof: Let as firstly assume that a zero sum assumption holds. Let us calculate expectation of D_P using it's distribution explained in (1).

$$\begin{aligned} E(D_P) &= -U(1-p) + p(kU - U) = \\ &= -U + Up + pkU - pU = \\ &= U(kp-1) \end{aligned}$$

Since a zero sum assumption holds, we get $U(kp-1) = 0$ and $k = 1/p$.

On the other hand, if we put $k = 1/p$ one can easily see that $E(D_P) = 0$ which means that a zero sum assumption holds. This shows that the zero sum assumption is equivalent to $k = 1/p$. □

Now, as announced in an Introduction, we will interpret a position of a bookmaker using a random variable D_B. That means that we will describe odds and probabilities in a style similar to a previous result.

Theorem 2.2. *Let k be a betting odd and let p be a probability of a bettor's win, then if a zero sum assumption holds, a distribution of a random variable D_B is given by*

$$D_B = \begin{pmatrix} \frac{1}{pq}U - \frac{1}{p}U & -\frac{1}{p}U \\ q & p \end{pmatrix} \tag{3}$$

where $q = 1 - p$.

Proof: We begin our proof by putting a bookmaker in a same position as a bettor. We are not making any biased assumptions by doing that. It means that we treat a bookmaker's exposure to a risk as if he had placed a bet where he might lose amount kU (if a bettor wins) or gain U (if a bettor lose). From that context, let us introduce a betting odd k_1 associated to a bookmakers 'bet'. In a case that a bookmaker wins, then clean profit should be $k_1(kU) - kU$. The reason is that a bookmaker invested kU, that is how much a bookmaker was exposed to a risk, and based on that a bookmaker receives award that is equal to $k_1(kU)$. Hence, a clean profit is $k_1(kU) - kU$. So, one possible value for D_B is

$$D_B = k_1(kU) - kU.$$

Since p was defined as a probability that a bettor will win, we can calculate a probability that a bookmaker will make profit. To be more precise we have:

$$P(D_B = k_1(kU) - kU) = 1 - p = q.$$

The case when a bookmaker lose money can be described by putting $D_B = -kU$. It means that a bettor won, hence a probability for that event is p. Thus

$$P(D_B = -kU) = p.$$

Altogether, we have:

$$D_B = \begin{pmatrix} k_1(kU) - kU & -kU \\ q & p \end{pmatrix} \tag{4}$$

A zero sum assumption yields $E(D_B) = 0$. This means that

$$E(D_B) = (k_1(kU) - kU)(1-p) - pkU = 0.$$

Using $pk = 1$ we get $U(k_1k - k)(1-p) = U$. This yields $k_1k - k = \frac{1}{1-p}$. From here we determine:

$$k_1k = k + \frac{1}{1-p} = \frac{1}{p} + \frac{1}{1-p} = \frac{1}{p(1-p)}.$$

Again, from $k = \dfrac{1}{p}$ we get $k_1 = \dfrac{1}{1-p} = \dfrac{1}{q}$. Therefore, $k_1kU - kU = \dfrac{1}{pq}U - \dfrac{1}{p}U$ and

$$D_B = \begin{pmatrix} \frac{1}{pq}U - \frac{1}{p}U & -\frac{1}{p}U \\ q & p \end{pmatrix} \tag{5}$$

□

3 Multiplication of odds for a player

This section contains a description of a case when a bettor place bet on more than one game. It means that a bettor will make profit only in the case when outcome of every game is as bettor has predicted. If a bettor missed at least one game, all of his investment is lost. As an adequate award for such a risk odds are multiplied.

Our goal is to provide an interpretation why odds are multiplied. As it will be shown, multiplication of odds will be direct consequence of a zero sum assumption. Additionally, we will see that if we assume that odds are multiplied, then we will have a conclusion that a zero sum assumption holds, hence those two concepts are mathematically equivalent.

Let say that a bettor place a bet on n games, where each game is made of pairs of teams, home team and away team. We could denote those pairs as: $G_1,\ G_2,\ \ldots, G_n$. Let p_i denote a probability that a bettor will guess an outcome of a game G_i, then a bettor will make a profit with a probability $p_1 \cdot p_2 \cdot \cdots \cdot p_n$ (he needs to guess each of n games correctly). Let say that a bettor invested amount U. We will say that k is accumulated odd if a bettor made a correct guess of all n games. Then a bettor receives award kU. We summarize results in following:

Theorem 3.1. *Let a bettor place a bet on n games $G_1, \ldots, G_n$, where p_i is a probability of a correct guess for a game G_i with odds k_i. Let k be accumulated odd and U is investment. Then a random variable D_P is given by:*

$$D_P = \begin{pmatrix} -U & kU - U \\ 1 - p_1p_2\cdots p_n & p_1p_2\cdots p_n \end{pmatrix}. \tag{6}$$

Additionally, the zero sum principle is equivalent to $k = k_1k_2\cdots k_n$.

Proof: If a bettor has a correct guess for all of n games, after investing amount U, a clean profit is $kU - U$, where k is accumulated odd. Probability for that event is $p_1p_2\cdots p_n$, therefore we can write $P(D_P = kU - U) = p_1p_2\cdots p_n$. A probability of an opposite is: $P(D_{ig} = -U) = 1 - p_1p_2\cdots p_n$. Assuming that a zero sum principle holds, we get $E(D_P) = 0$. Therefore

$$E(D_P) = -U(1 - p_1p_2\cdots p_n) + p_1p_2\cdots p_nU(k-1) = 0.$$

From here, we get $-1 + p_1p_2\cdots p_n + p_1p_2\cdots p_nk - p_1p_2\cdots p_n = 0$. Therefore, $1 = p_1p_2\cdots p_nk$ and finally

$$k = \frac{1}{p_1p_2\cdots p_n}.$$

By a previous theorem, we have $k_i = 1/p_i$. Hence

$$k = \frac{1}{p_1p_2\cdots p_n} = \frac{1}{p_1}\cdot\frac{1}{p_2}\cdots\frac{1}{p_n} = k_1k_2\cdots k_n.$$

This means that accumulated odd is equal to a product of each respective odd for each of n games.

Using similar approach from $k = k_1k_2\cdots k_n$ we can get $E(D_P) = 0$. □

4 Different rule for the position of a bookmaker

This section deals with, in some sense, inverse approach regarding a position of a bookmaker. To be more precise, we will treat a bookmaker as a bettor. That means that a bookmaker is also exposed to a risk, like a bettor. Also, if outcome of games $G_1, \ldots, G_n$ are not in a bettor favour, then a bookmaker will gain a profit and a bettor will lose all of his investment. Therefore, we can also treat a bookmaker as a 'bettor.' Thus, in order to provide further interpretations, we shall define odds that are associated with a bet that a bookmaker in obviously making when accepting a risk and taking a bet from a bettor. As we will see later in the section, there is one counterintuitive consequence of the zero sum principle. Let K represent odds for a bookmaker to make a profit.

As it will be demonstrated, the odds from the perspective of the bookmaker do not multiply; they follow a different pattern. We will also use random variable D_B that describes clean profits and losses that a bookmaker might have. We summarize our results as follows:

Theorem 4.1. *If a bettor place a bet on n pairs, where probability of a win for G_i is p_i and odd for a same pair is k_i, then a zero sum principle implies that bookmakers odds are given by*

$$K = \frac{1}{1-P} = \frac{1}{Q}$$

where $P = p_1p_2\cdots p_n$ and $Q = 1 - P$. The random variable D_B is given by:

$$D_B = \begin{pmatrix} \frac{1}{PQ}U - \frac{1}{P}U & -\frac{1}{P}U \\ Q & P \end{pmatrix} \tag{7}$$

The inverse of this theorem holds as well.

Proof: Coefficient K is a measure of the bookmaker's award for an investment of kU, because kU is the amount that is exposed to a risk from the bookmaker's point of view. We must subtract kU, because that is the amount already owned by the bookmaker. Therefore, this could be stated as $D_B = K(kU) - kU$. In the case of the bookmaker's lost, we have $D_B = -kU$ and

$$P(D_B = K(kU) - kU) = 1 - p_1 p_2 \cdots p_n = 1 - P,$$

$$P(D_B = -kU) = p_1 p_2 \cdots p_n = P.$$

Hence, we have the following distribution for the bookmaker:

$$D_B = \begin{pmatrix} K(kU) - kU & -kU \\ 1 - P & P \end{pmatrix} \tag{8}$$

Assuming a zero sum principle, we have

$$E(D_B) = (1 - P)(K(kU) - kU) - P \cdot \frac{1}{P} U = 0.$$

We already know that $k = k_1 \cdots k_n = 1/(p_1 \cdots p_n) = 1/P$. Hence, $kU(K - 1) = U/(1 - P)$. This gives us $K = 1/(1 - P) = 1/Q$. Finally,

$$D_B = \begin{pmatrix} \frac{1}{PQ}U - \frac{1}{P}U & -\frac{1}{P}U \\ Q & P \end{pmatrix} \tag{9}$$

To summarize, based on the zero sum principle, we can multiply the betting odds for a player, but if try the same for the bookmaker we get to a conclusion that each betting odd for a pair i is $1/(1 - p_i)$. Thus a product of odds is

$$\frac{1}{1 - p_1} \cdot \frac{1}{1 - p_2} \cdot \frac{1}{1 - p_n}.$$

On the other hand we see that

$$\frac{1}{1 - p_1} \cdot \frac{1}{1 - p_2} \cdot \frac{1}{1 - p_n} \neq K = \frac{1}{1 - P} = \frac{1}{1 - p_1 \cdots p_n}.$$

So, from a position of a bookmaker odds don't multiply in a same sense like in a bettor's case. $\square$

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